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# Numerical simulations of transitional flows using $\gamma$ -SST model

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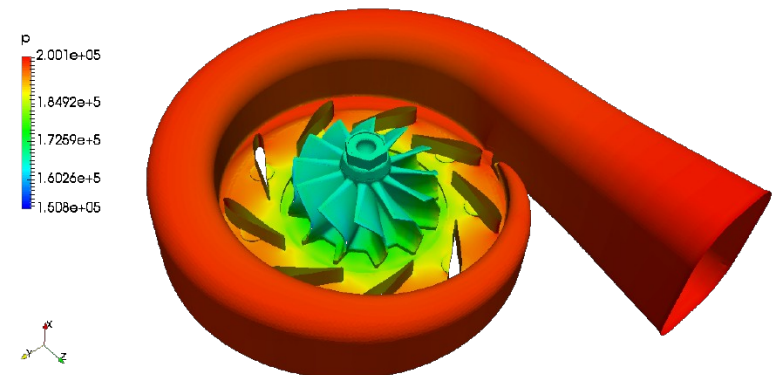
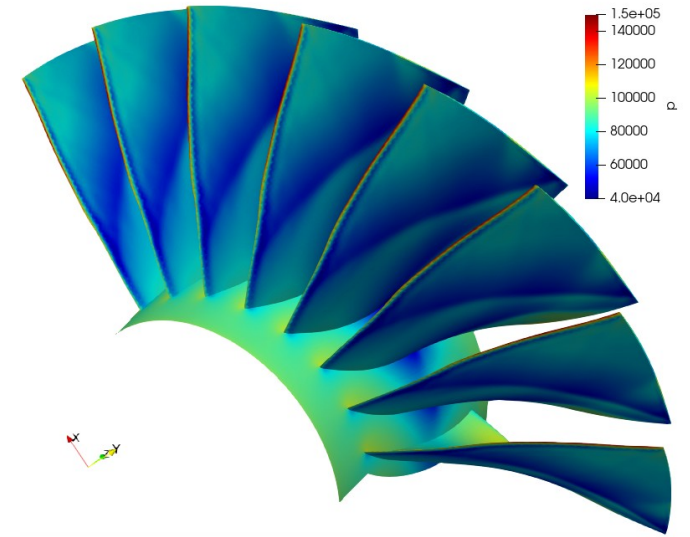
# Introduction

## Turbomachinery flows:

- High speed, compressible internal flows
- Transonic regimes, shock waves, ...
- Rotating system, multiple frames of reference,
- Specific boundary conditions (stator-rotor interfaces, non-reflecting boco, ...)
- Complicated physics (EOS, non-equilibrium condensation of steam, ...)
- Surface roughness

## Typical flow regimes:

- $Ma = 0.5 - 2$
- $Re \sim 10^6$
- $Tu = 2 - 10\%$
- Bypass transition



# Mean flow solver

## OpenFOAM package

- An open source framework for finite-volume method
- Contains several ready made solvers for incompressible flows, heat transfer, ...
- OpenFOAM solvers for high-speed compressible flows:
  - × *rhoSimpleFoam* – segregated, very “fragile”, sensitive to under-relaxation, ...
  - × *rhoCentralFoam* – coupled, explicit time stepping (not suitable for Low-Re approach)

## In-house solver [1]

- Built on top of OpenFOAM package
- standard FVM with convective fluxes evaluated using approximate Riemann solvers (HLLC with low-Mach correction [2])
- Second order in space using piece-wise linear reconstructions with limiters
- Implicit time stepping with matrix-free LU-SGS method
- Weak coupling with transition and turbulence model:
  1. Update mean flow variables ( $\rho, U, E$ ) using old values of turbulence variables ( $k, \omega, \gamma, \dots$ )
  2. Update turbulence model using new mean flow variables (with under-relaxation)

Solver available at: <https://github.com/furstj/myFoam>

[1] FÜRST, Jiří. *Development of a coupled matrix-free LU-SGS solver for turbulent compressible flows*. *Computers & Fluids*. August 2018. Vol. 172, p. 332–339. DOI 10.1016/j.compfluid.2018.04.020.

[2] XIE, Wenjia, ZHANG, Ran, LAI, Jianqi and LI, Hua. *An accurate and robust HLLC-type Riemann solver for the compressible Euler system at various Mach numbers*. *International Journal for Numerical Methods in Fluids*. 2019. DOI 10.1002/fld.4704.

# Transition and turbulence model

## $\gamma$ -SST model [3]

- Local correlation based model, similar to Langtry's & Menter's  $\gamma$ - $Re_\theta$  model with  $Re_\theta$  equation replaced by an algebraic relation
- $\gamma$  equation is coupled to SST model with  $P_k$  calculated using Kato-Launder modification
- Contains additional source term for transition at low  $Tu$  and for transition in separated flows [3]
- Missing: reliable cross-flow transition, compressibility effects, ...

## $k$ - $k_L$ - $\omega$ model [4]

- Three-equation laminar kinetic energy model
- Suitable for bypass and natural transition
- Missing: separation induced transition, cross-flow transition, compressibility effects, ...

Both models available as an additional library for OpenFOAM at:

<https://github.com/furstj/myTurbulenceModels>

[3] MENTER, Florian R., SMIRNOV, Pavel E., LIU, Tao and AVANCHA, Ravikanth. A one-equation local correlation-based transition model. Flow, Turbulence and Combustion. 5 December 2015. Vol. 95, no. 4, p. 583–619. DOI 10.1007/s10494-015-9622-4

[4] LOPEZ, Maurin and KEITH WALTERS, D. A Recommended Correction to the  $k_T - k_L - \omega$  Transition-Sensitive Eddy-Viscosity Model. Journal of Fluids Engineering [online]. 7 December 2016. Vol. 139, no. 2, p. 024501. DOI 10.1115/1.4034875.

# Results: Flat plate flows

## Case 1A

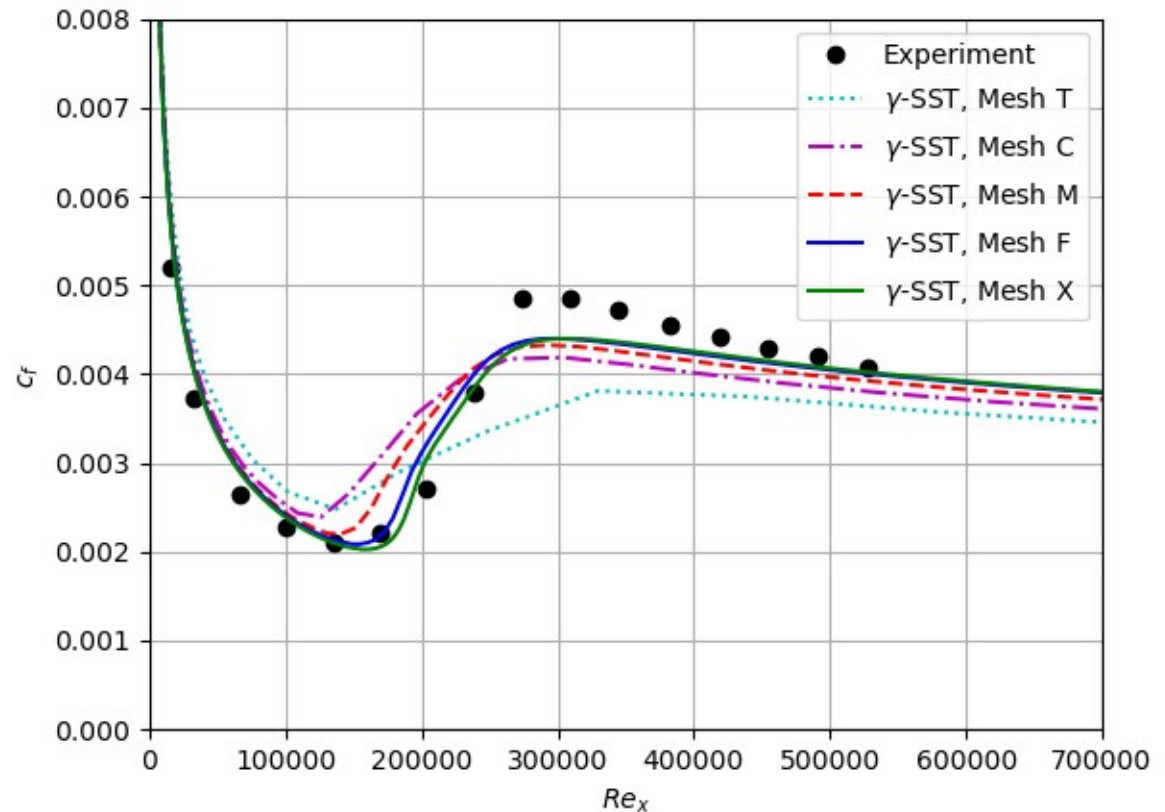
Computation:

- $Tu = 5.86\%$  (at inlet)
- $\mu_i/\mu = 11.9$

Experiment (ERCOFTAC)

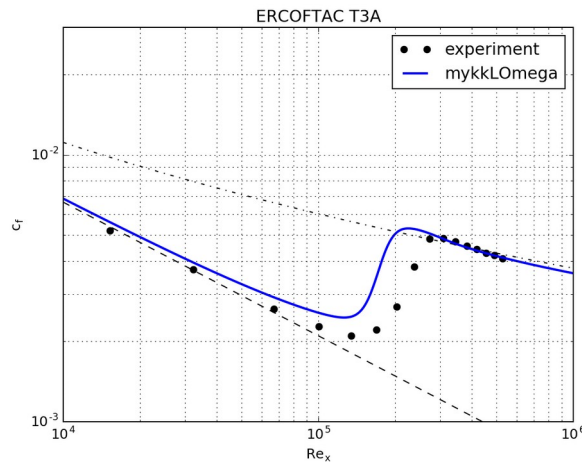
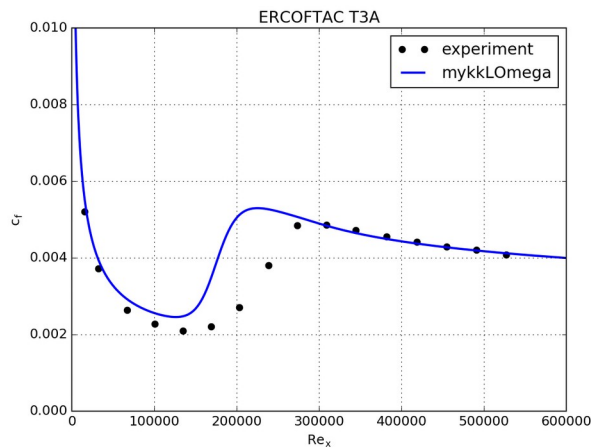
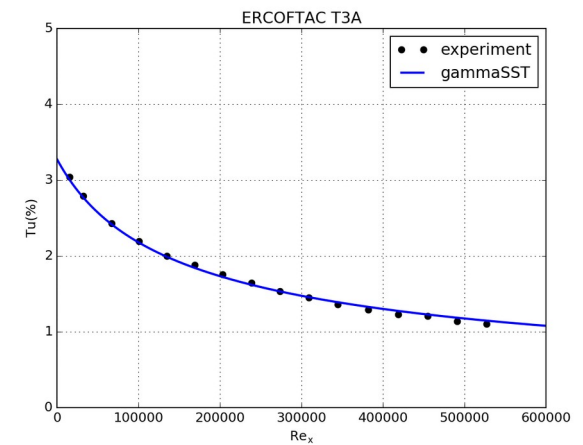
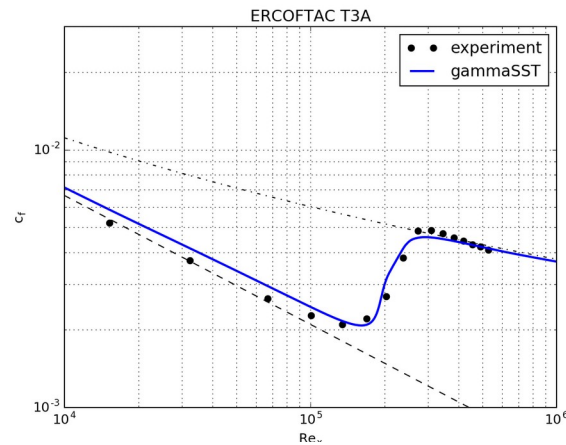
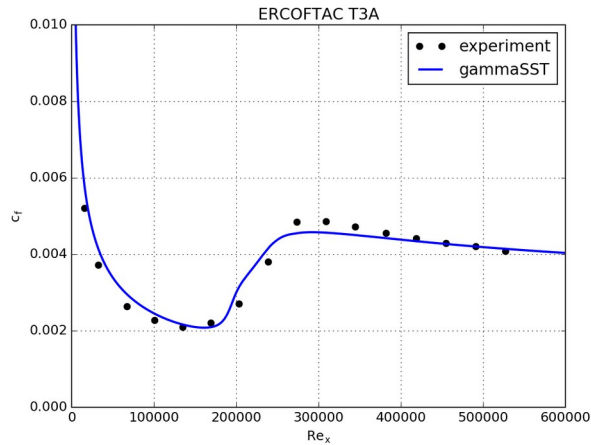
- $Tu = 3.3\%$  (at LE)

Mesh	# cells	$y^+$
Tiny	1 056	1.50
Coarse	4 224	0.68
Medium	16 896	0.32
Fine	67 584	0.16
eXtra f.	270 336	0.08



# Results: Flat plate flows

## Case 1A, custom mesh & BC



Mesh:

- 66 675 cells,
- $y^+ \sim 0.2$ ,
- similar to mesh F

Incompressible fluid

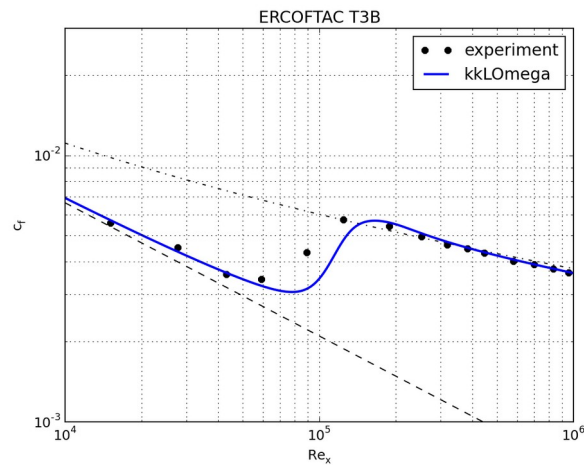
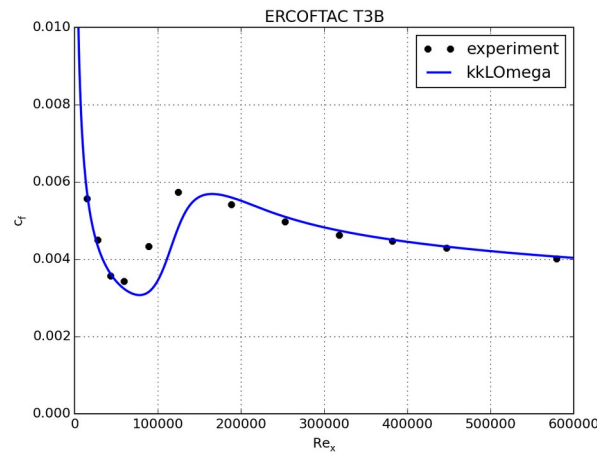
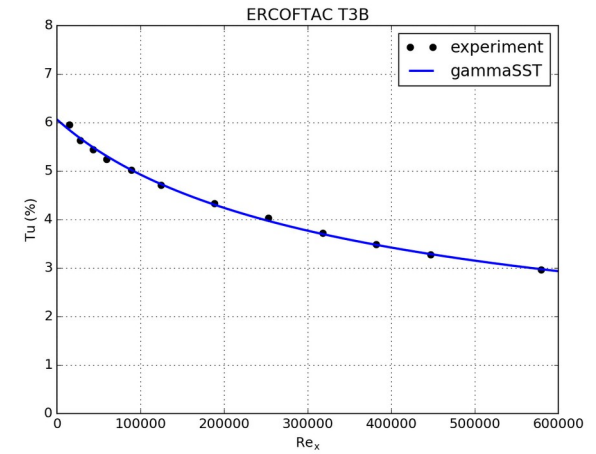
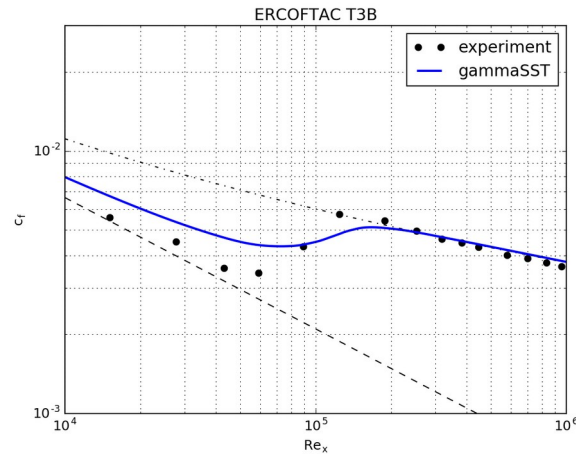
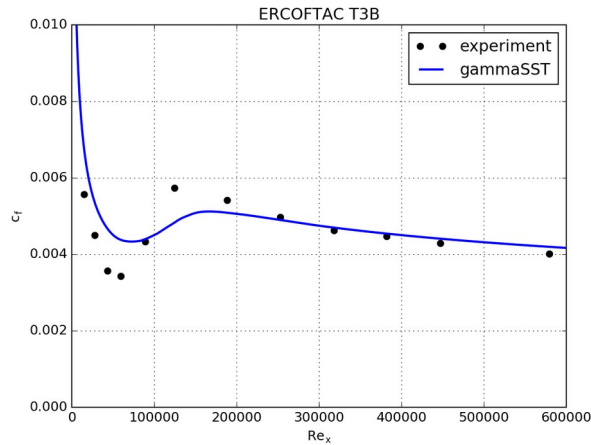
SIMPLE algorithm

$Tu = 3.7\%$

$\nu_t/\nu = 12.3$  (for  $\gamma$ -SST)

# Results: Flat plate flows

## Case 1B, custom mesh & BC



Mesh:

- 66 675 cells,
- $y^+ \sim 0.2$ ,
- similar to mesh F

Incompressible fluid

SIMPLE algorithm

$Tu = 6.6 \%$

$\nu_t/\nu = 100$  (for  $\gamma$ -SST)



# Results: NLF(1)-0416

Data for  $Tu=0.15\%$  : submitted TWS

Here: data for

- $Re = 4\,000\,000$
- $M = 0.1$
- $Tu = 0.11\%$
- $\omega_\infty = 5 U / \text{chord}$

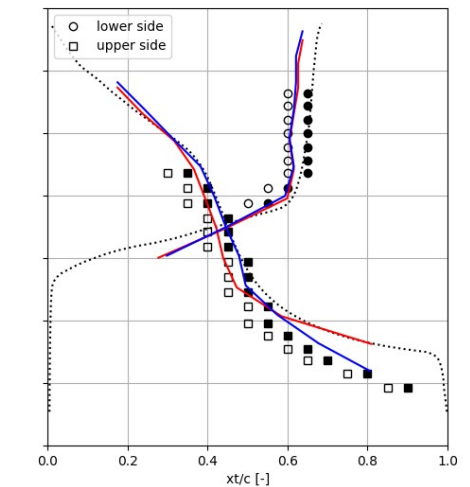
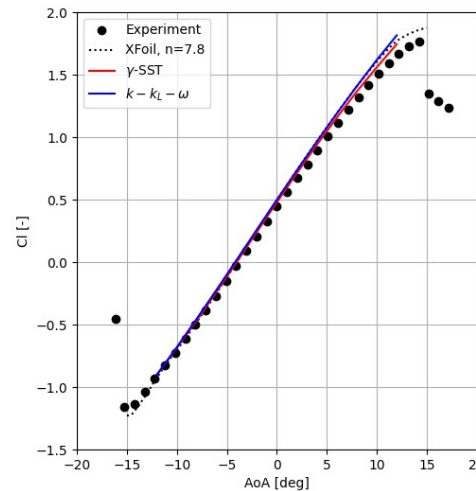
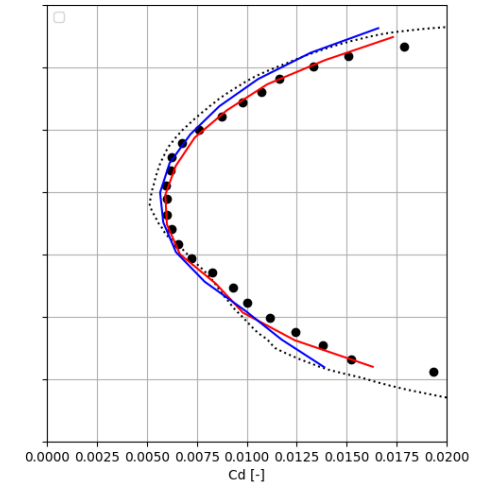
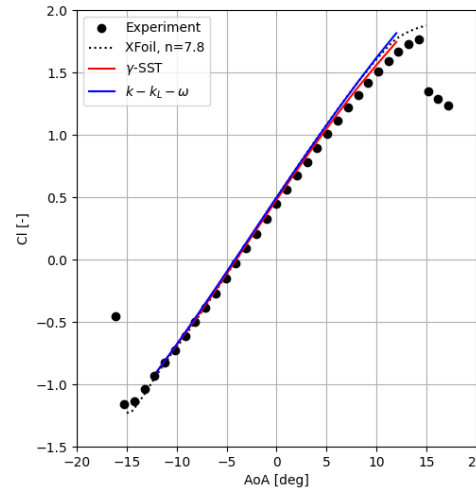
Mesh:

- 67 584 cells
- $y^+ \sim 0.2$

Comparison with:

- XFOIL ( $n = 7.8$ )
- Experimental data [5]

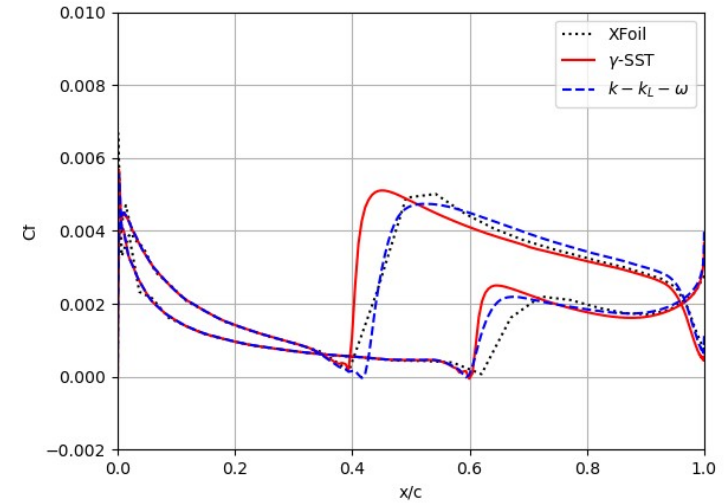
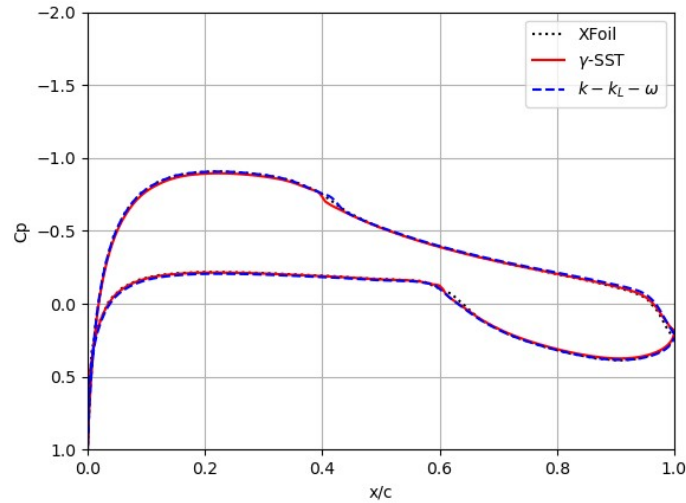
[5] Somers D.: Design and experimental results for a natural laminar flow airfoil for general aviation applications, NASA-TP-1861, 1981



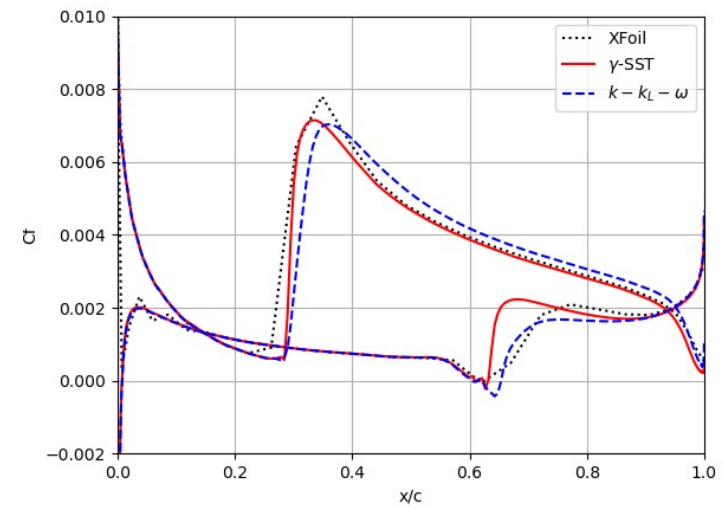
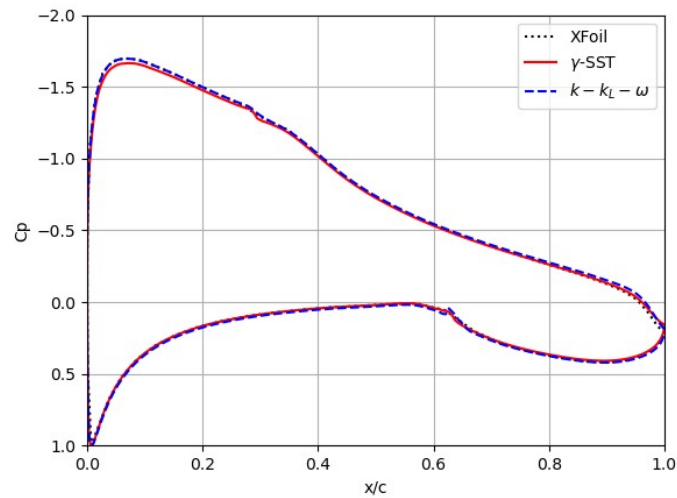


# Results: NLF(1)-0416

AoA = 0°

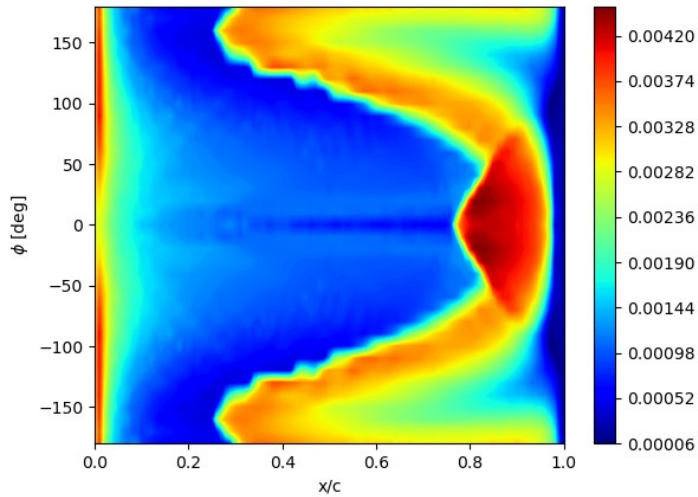
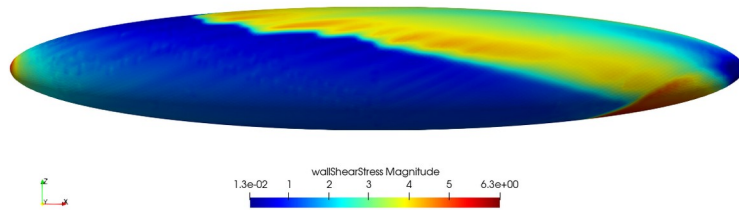


AoA = 5°

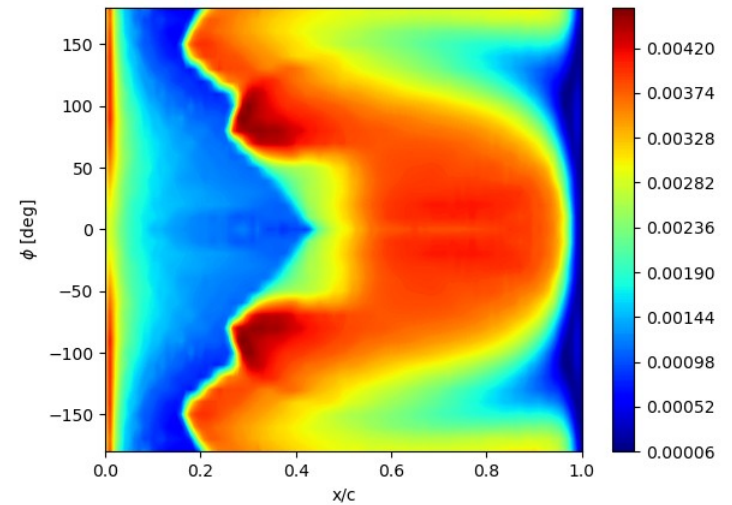
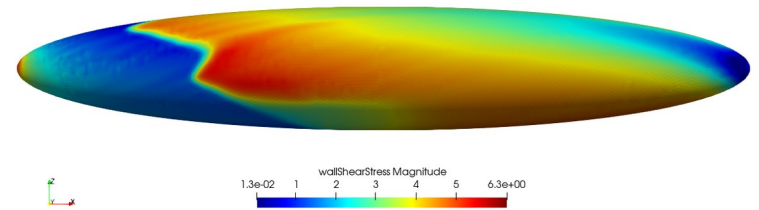


# Results: Ellipsoid

## Case 3, $\text{AoA} = 10^\circ$



without CF correlation



with CF correlation (C1)

# Results: Case 4

$AoA = 1.98^\circ$

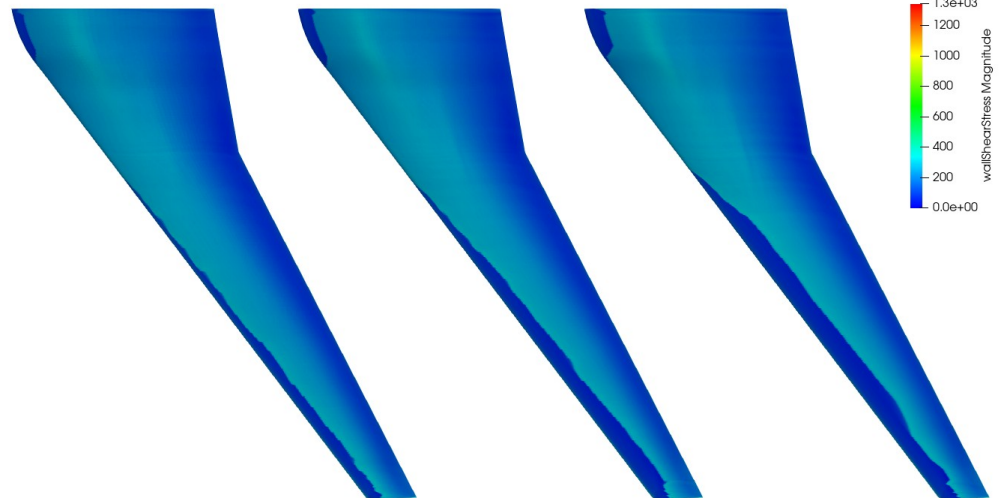
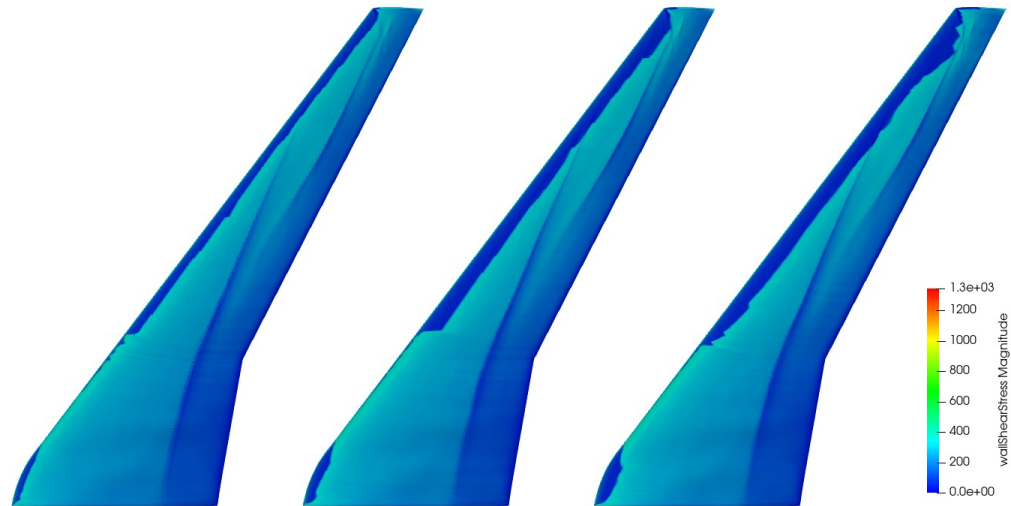
- Unstructured mesh (Series A)
- gamma-SST model
- No CF correlation
- No sustaining term

## Mesher:

8 – 7 mil. elems

12 – 20 mil. elems

16 – 46 mil. elems



Mesh 8

Mesh 12

Mesh 16

Mesh	Cl	Cd
8	0.3826	0.0234
12	0.3859	0.0220
16	0.3890	0.0215

# Conclusion

## Results obtained with $\gamma$ -SST model

- Case 1 & 2 – quite good results consistent with experiment and other models
- Case 3 (ellipsoid) – very bad results due to missing CF correlation
- The  $\gamma$ -SST model needs improvements for
  - × cross-flow transition (C1 is valid only for wings)
  - × compressibility, shock-wave BL interaction,
  - × transition due to distributed roughness (some work already done for  $\gamma$ - $Re_\theta$  by Dassler et al. or Langel et al.)

## **Acknowledgements:**

Authors acknowledge support from the Center of Advanced Applied Sciences CZ.02.1.01/0.0/0.0/16\_019/0000778.

Computational resources were supplied by the project "e-Infrastruktura CZ" (e-INFRA LM2018140) provided within the program Projects of Large Research, Development and Innovations Infrastructures.